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Analysis and Relevance of Z-Transform in Discrete Time Systems

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Abstract

This research paper presents the analysis and relevance of z-transform in discrete time systems. The computer implementations of majority of engineering and physical systems require discretization of continuous parameters such as time, frequency, temperature, etc. Such systems are called discrete time systems and their dynamics can be described as reoccurrence equations. Many questions have been raised on the relevance of z-transform in digital signal processing. The study was conducted to show the analysis and relevance of z-transform in discrete time system. Z-transform is a mathematical too to transform a time-domain mode to a corresponding complex-frequency domain mode. We present different equations of z-transform along some its properties such as linearity, time delay and complex translation. An interesting part of the work is the uniqueness of z-transform and all the z-transform values must converge for adequate signal transmission to avoid signal degradation.

Keywords: Signal, processing, digital, z-transform, domain.

Introduction

Digital signal processing (DSP) is the use of digital processing, such as by computers or more special digital signal processors, to perform a wide variety of signal processing operations. (1) The signals processed in this manner are a sequence of numbers that represent samples of a continuous variable in a domain such as time, space, or frequency when adequately sampled. DSP can involve linear or nonlinear operations. Nonlinear signal processing is closely related to nonlinear system identification [1] and can be implemented in the time, frequency, and spatio_temporal domains. The application of digital computation to signal processing allows for many advantages over analog processing in many applications, such as error detection and correction in transmission as well as data compression (2). DSP is applicable to both streaming data and static (stored) data.

Signal Sampling

To digitally analyze and manipulate an analog signal, it must be digitized with an analog-to-digital converter (ADC) [9]. Sampling is usually carried out in two stages, discretization and quantization. Discretization means that the signal is divided into equal intervals of time, and each interval is represented by a single measurement of amplitude. Quantization means each amplitude measurement is approximated by a value from a finite set. Rounding real numbers to integers is an example. The Nyquist_Shannon sampling theorem states that a signal can be exactly reconstructed from its samples if the sampling frequency is greater than twice the highest frequency component in the signal. In practice, the sampling frequency is often significantly higher than twice the Nyquist frequency. (3)

Theoretically, digital signal processing analysis and derivations are typically performed on discrete-time signal models with no amplitude inaccuracies (quantization error), created by the abstract process of sampling. Numerical methods require a quantized signal, such as those produced by an analog digital converter. The processed result might be a frequency spectrum or a set of statistics. But often it is another quantized signal that is converted back to analog form by a digital-to-analog converter.

Domains

In digital signal processing, engineers usually study digital signals in one of the following domains: time domain (one-dimensional signals), spatial domain (multidimensional signals), frequency domain, and wavelet domains. They choose the domain in which to process a signal by making an informed assumption (or by trying different possibilities) as to which domain best represents the essential characteristics of the signal and the processing to be applied to it. A sequence of samples from a measuring device produces a temporal or spatial domain representation, whereas a discrete Fourier transform produces the frequency domain representation. (3)

Applications of Digital Signal Processing:

There are various applications of digital signal processing in our society. (8) They include

- **Speech Processing**: Speech is one dimensional signal. Digital processing of speech is applied to a wide range of speech problems such as speech spectrum analysis, channel voice decoders, etc. DSP is also applied in speech encoding, speech enhancement and speech analysis and synthesis.
- **Image Processing**: Any two dimensional pattern is called an image. Digital processing of images requires two dimensional DSP tools such as; Discrete Fourier Transform (DFT), Fast Fourier Transform (FFT), algorithms and Z_transforms. Processing of electrical signals extracted from images b digital techniques include: image formation and recording, image compression, image restoration, image reconstruction and image enhancement.
- **Radar Signal Processing**: Radar stands for radio detection and ranging. Improvement in signal processing is possible in digital technology. Development in DSP as lead to greater sophisticated of radar tracking algorithms consisting of transmit receive antenna, digital processing system and control unit.
- **Digital Communications**: Application of DSP in digital communication, especially telecommunications comprise of digital transmission using PCM digital switching TD, eco control and digital tape, recorders. DSP in telecomm are found to be cost effective due to availability of medium and large scale digital ICs.
- **Spectral Analysis**: Frequency-domain analysis is easily and effectively possible in DSP using FFT algorithms. These algorithms reduce computational complexity and time.
- **Sonar Signal Processing**: Sonar stands for second navigation and ranging. Sonar is used to determine the range, velocity and direction of targets that are remote from the observers.

Other applications include: transmission line, advanced optical fiber communication, analysis of sound and vibration signals, satellite communications, telephony transmission, aviation, astronomy, microprocessor systems, industrial noise control, etc.

Z_Transform

The z_transform is useful for the manipulation of discrete data sequences and has acquired a new significance in the formulation and analysis of discrete-time systems. It is used extensively today in the areas of applied mathematics, digital signal processing, control theory, population science, and economics. These discrete models are solved with difference equations in a manner that is analogous to solving continuous models with differential equations. The role played by the z-transform in the solution of difference equations corresponds to that played by the Laplace transforms in the solution of differential equations. (5)

Z-transform, like the Laplace transform, is an indispensable mathematical tool for the design, analysis and monitoring of systems. The z_transform is the discrete_time counter-part of the Laplace transform and a generalization of the Fourier transform of a sampled signal. Like Laplace transform the z_transform allows insight into the transient behavior, the steady state behavior, and the stability of discrete-time systems. A working knowledge of the z-transform is essential to the study of digital filters and systems. This paper begins with the definition of the derivation of the z_transform from the Laplace transform of a discrete-time signal. A useful aspect of the Laplace and the z-transforms are there presentation of a system in terms of the locations of the poles and the zeros of the system transfer function in a complex plane. (4)

The z-transform is a powerful tool for solving different equations and is a generalization of the Discrete-Time Fourier Transform. It is used because the DTFT does not converge/exist for many important signals, and yet does for the z_transform. It is also used because it is cleaner than the DTFT. In contrast to the DTFT, instead of using complex exponentials of the form e^{jwn}, with purely imaginary parameters, the Z transform uses the more general, Z^{n} where z is complex. Whenever you are talking about z-transform, you must the region of convergence **(ROC).** The region of convergence, known as the ROC, is important to understand because it defines the region where the z-transform exists. The Z-transform thus allows one to bring in the power of complex variable theory into Digital Signal Processing. (8) The ROC for a given $x[n]$, is defined as the range of z for which the z-transform converges. Since the z-transform is a power series, it converges when $X(n)z^{-n}$ is absolutely summable. The z-transform equation is given thus (6):

Fourier Transform:

$$\boldsymbol{X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n)\, e^{j\omega}} \qquad \ldots\ldots\ldots\ldots\ldots..(\mathbf{1})$$

Z-Transform:

$$\boldsymbol{X(z) = \sum_{n=-\infty}^{\infty} x(n)\, Z^{-n}} \qquad \ldots\ldots\ldots\ldots\ldots..(\mathbf{2})$$

This is regarded as two sided or bilateral of z-transform; observing that z is a complex variable. All relationship between a sequence and its z-transform is given by:

$$\mathbf{X(n)} \xleftrightarrow{z} \mathbf{X(z)}$$

Analyzing the above equations, the series must converge for all values of z, except z=0 and z=∞. When x-rayed for values of X(n) = (1,2,3,4) using equation 2;

$$X(z) = \sum x(n)\, z^{-n} = \sum x(n)\, z^{-n}$$

Note that x(n) exists from o to 4, hence n has limits from 0 to 4, using application of power series:

$$X(z) = X(0)\, Z0 + X(1)\, Z^{-}1 + X(2)\, Z^{-2} + X(3)\, Z^{-3} + X(4)\, Z^{-4}$$

Substituting the values of X(0), X(1), X(2), X(3), and X(4) in the equation given;

$$X(z) = 1 + 2z^{-1} + 3z^{-2} + 4z^{-3}$$

Hence, the X(z) must converge for all values except for z=0. The ROC will result in the entire pane except at z = 0 in the figure below. The z-transform is an infinite power series and need to converge at very critical sequences; hence for every

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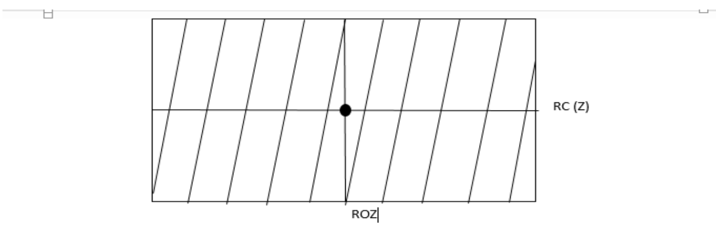
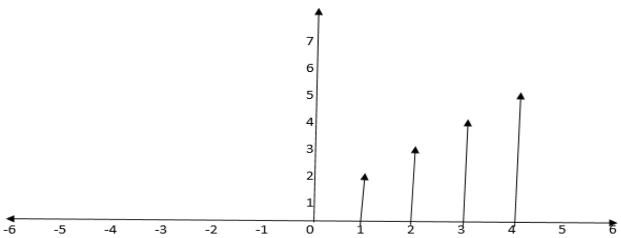
set of sequence, its ROC must be different. [6]





Properties of Z-Transform

We have various properties of z-transform in digital signal processing. [6] They include:

1. Linearity Property:

Mathematically, it states that: $X_1(n) \overset{z}{\longleftrightarrow} X_1(z)$ ROC R_1

$X_2(n) \overset{z}{\longleftrightarrow} X_2(z)$ ROC R_2

$Y(n) = Ax_1(n) + Bx_2(n) \overset{z}{\longleftrightarrow} Ax_1(z) + Bx_2(n)$ ROC R_1 n R_2

2. Time Differentiation Property:

Mathematically, it states that: $X(n) \overset{z}{\longleftrightarrow} X(z)$ ROC: R

$Y(n) = nX(n) \overset{z}{\longleftrightarrow} Y(z) = d_{/dz}\, X(z)$ ROC: R

3. Convolution Property:

Mathematically, it states that: $X_1(n) \overset{z}{\longleftrightarrow} X_1(z)$ ROC:R_1

$X_2(n) \overset{z}{\longleftrightarrow} X_2(z)$ ROC:R_2

$Y(n) = X_1(n) * X_2(n) \overset{z}{\longleftrightarrow} Y(z) = X_1(n)\, X_2(n)$ ROC: R_1 n R_2

4. Shifting Property:

Mathematically, it states that: $X(n) \overset{z}{\longleftrightarrow} X(z)$ ROC: R

$Y(n) = X(n-k) \overset{z}{\longleftrightarrow} Y(z) = Z^{-k} X(z)$ ROC: R n $[0 \leq |z| \leq \infty]$.

Inverse Z-Transform

This is of great importance in the discrete time linear systems evaluation. It is used in finding the impulse response H(n) of a digital filter from the given system H(z). This paces at advantage of other transformation. A system with $X(z) = 1 + 3z^{-1} - 2Z^{-2} + 4Z^{-3}$, assuming that $X(n) = 0; n \leq 0$ hence: X(n) = (1,3,-2,4). Inverse z-transform can resolve these issues in different ways which include:

- Long division or partial fraction.
- Residual method.

Admissive Form of Z_Transform

Formulas for $X(z)$ do not arise in a vacuum. In an introductory course they are expressed as linear combinations of z-transforms corresponding to elementary functions such as

$S = \{\delta[n],\ u[n],\ b^n,\ e^{an},\ n,\ n^2,\ n^m,\ n b^n,\ n e^{an},\ b^n \operatorname{Sin}[a n],\ b^n \operatorname{Cos}[a n],\ \operatorname{Sinh}[a n],\ \operatorname{Cosh}[a n],$

It can be shown that a linear combination of rational functions is a rational function. Therefore, for the examples and applications considered in this study, we can restrict the z-transforms to be rational functions. This restriction is emphasized this in the following definition.

Given that the z-transform $X(z) = \sum_{n=0}^{\infty} x[n] z^{-n}$, we say that $X(z)$ is an admissible z-transform, provided that it is a rational function, that is

$$X(z) = \frac{P(z)}{Q(z)} = \frac{b_0 + b_1 z^1 + b_2 z^2 + \ldots + b_{p-1} z^{p-1} + b_p z^p}{a_0 + a_1 z^1 + a_2 z^2 + \ldots + a_{q-1} z^{q-1} + a_q z^q}$$

,

where $P(z)$ and $Q(z)$, are polynomials of degree p and q, respectively.

From our knowledge of rational functions, we see that an admissible z-transform is defined everywhere in the complex plane except at a finite number of isolated singularities that are poles and occur at the points where $Q(z) = 0$. The Laurent series expansion can be obtained by a partial fraction manipulation and followed by geometric series expansions in powers of $\frac{1}{z}$ However, the signal feature of formula above is the calculation of the inverse z-transform via residues. (5)

Application of Z_Transform in Discrete Time System:

There are various ways where we apply the principle of z_transform in digital signal processing. They include:

1. It is used to analyze digital filters in discrete time system.
2. It is used to simulate the continuous systems.
3. It is used to analyze the linear discrete system.
4. It is used for automatic controls in telecommunication industry.
5. It helps in enhancing the electrical and mechanical energy to provide dynamic nature of the system.
6. It helps in system design and analysis, and also checks the system stability.

Conclusion:

In this paper, we reported the formal analysis of discrete time systems using the z-transform which is one of the most widely used transform methods in signal processing and communication engineering. This research work has been thoroughly explained and shown the beauties of z-transform application in solving different equations which form the bedrock of digital signal processing and digital filters operations. Its advantage over other transforms especially the inverse value of the transform is very high.

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